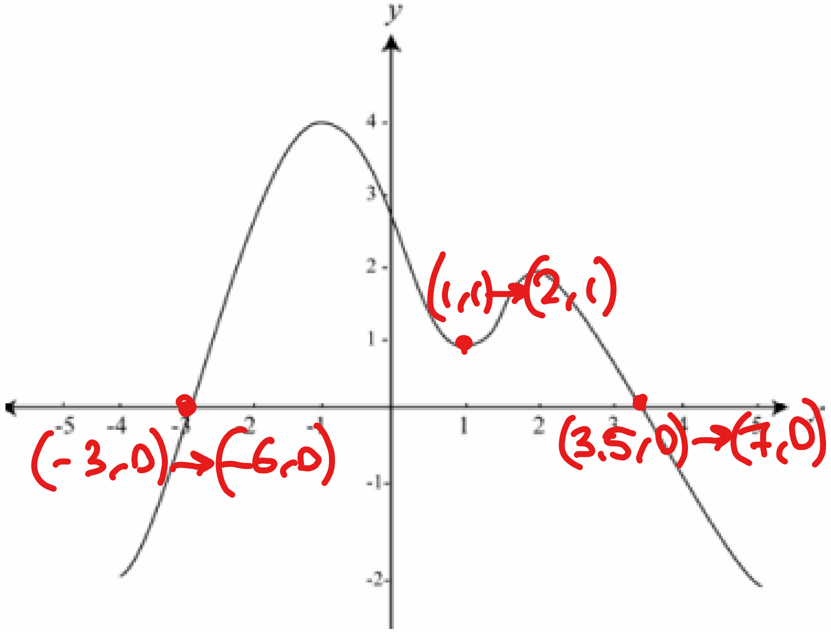
# Pre Calculus 12H
# Ch2 Transformation REVIEW

Key by Mahyar Pirayesh

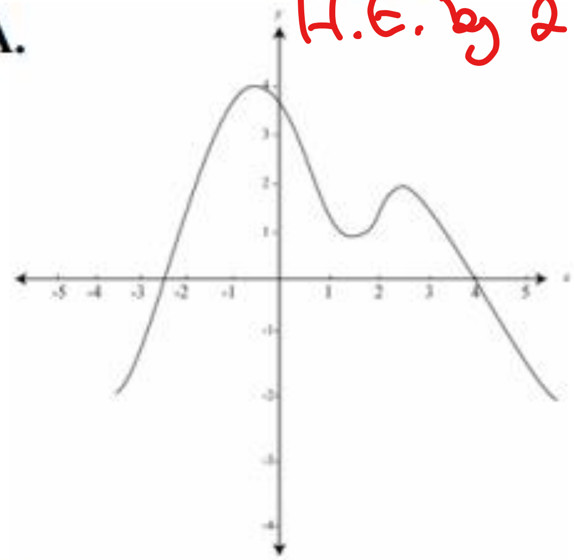
The graph of $y = f(x)$ is shown below



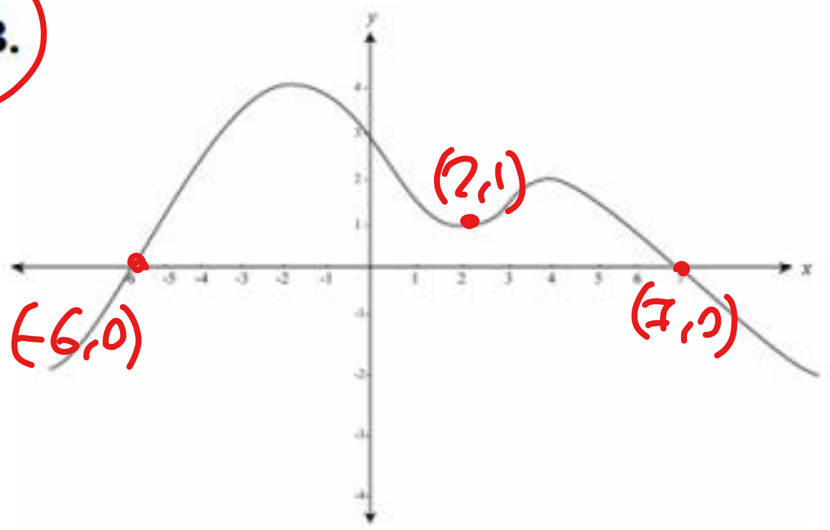

The graph of $f\left(\frac{1}{2}x\right)$ is correctly represented by which of the following?

H.E. by 2

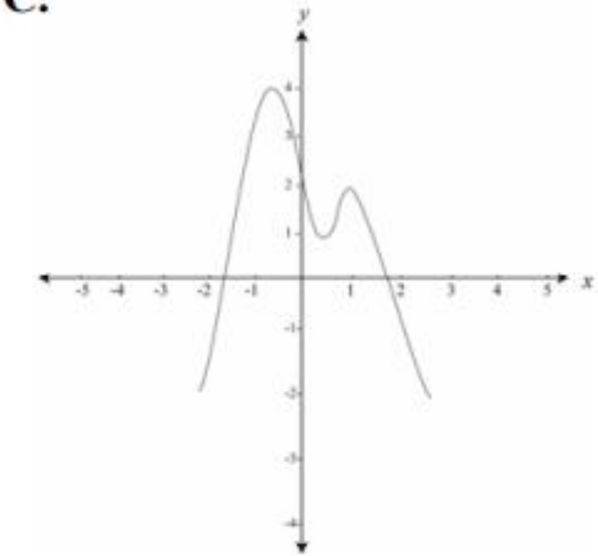
**A.**



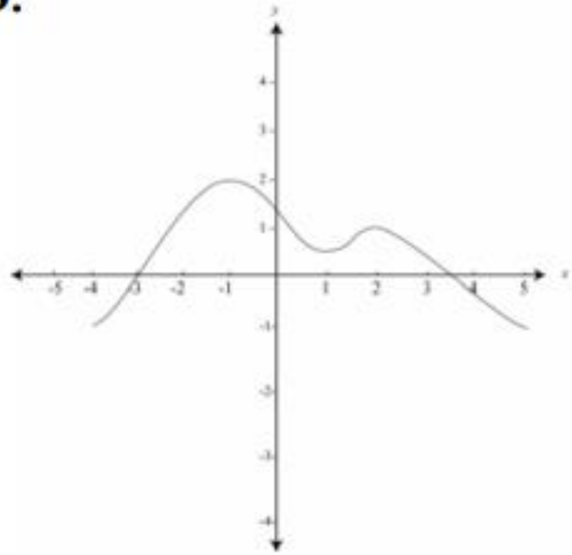
**B.**




**C.**



**D.**



If $y$ is replaced with $\frac{1}{3}y$ in the equation $y = f(x)$, then the resulting transformation on the graph will be

V.E. by 3 $y \to \frac{1}{3}y$

**A.** A vertical stretch by a factor of $\frac{1}{3}$ about the $x$-axis

**B.** A vertical stretch by a factor of 3 about the $x$-axis

**C.** A horizontal stretch by a factor of $\frac{1}{3}$ about the $y$-axis

**D.** A horizontal stretch by a factor of 3 about the $y$-axis

The graph of $f(x)=x^2-2$ undergoes the transformation $f(x+1)$.

If a student wishes to graph the transformed function in their calculator, the equation that gives the correct graph is

H.S. 1L $\quad x \to x+1$

$f(x+1) = (x+1)^2 - 2$

**A.** $x^2-1$

**B.** $x^2-3$

**C.** $(x+1)^2-2$

**D.** $(x-1)^2-2$

How many of the following functions will stay the same after a reflection over the y-axis?

$i)\, y=\sqrt{3}x^2+11$

$ii)\, y=-(x-3)^2+13$

$iii)\, y=2\sqrt{3x-4}$

$iv)\, y=\frac{-1}{x}+1$

$v)\, y=2|4x-3|$

$vi)\, y=3x^3+2x^2+1$

a) Only 1
b) Two Functions
c) Three Functions
d) Four Functions
e) ALL of THEM
f) None of them

Only i

$y=\sqrt{3}\,x^2+11 \rightarrow y=\sqrt{3}\,(-x)^2+11$
$=\sqrt{3}\,x^2+11$

The following function is Horizontally shifted 6 units LEFT. Which of the equations is the resulting function?

$$y=\sqrt{3x+2}-4$$

$$a)\ y=\sqrt{3x+8}-4$$

$$b)\ y=\sqrt{3x-4}-4$$

$$c)\ y=\sqrt{3(x-6)-4}-4$$

$$d)\ y=\sqrt{3x+14}-4$$

none

$y=\sqrt{3x+2}-4$

H.S. 6L $\quad x \to x+6$

$y=\sqrt{3(x+6)+2}-4$

$\boxed{y=\sqrt{3x+20}-4}$

The graph $y = f(x)$ contains the point (3, 4). After a transformation, the point (3, 4) is transformed to (5, 5). Which of the following is a possible equation of the transformed function?

**A** $y + 1 = f(x + 2)$
**B** $y + 1 = f(x - 2)$
**C** $y - 1 = f(x + 2)$
**D** $y - 1 = f(x - 2)$

$(3,4) \to (5,5)$

H.S. 2R $\quad x \to x-2$

V.S. 1U $\quad y \to y-1$

The graph of $y = |x|$ is transformed by a vertical stretch by a factor of 3 about the $x$-axis, and then a horizontal translation of 3 units left and a vertical translation up 1 unit. Which of the following points is on the transformed function?

A (0, 0)
B (1, 3)
C (−3, 1)
D (3, 1)

plug each in

V.E. by 3 $\quad y \to \frac{1}{3}y \quad y = 3|x|$

H.S. 3L $\quad x \to x+3 \quad y = 3|x+3|$

V.S. 1U $\quad y \to y-1 \quad y = 3|x+3|+1$

if $(x, y) = (-3, 1) \Longrightarrow 1 = 3|-3+3|+1$

$1 = 1$ ✓

The function below is shifted 1 unit left, then HE by 2, then HS 2 left, and then HE by 3. Which of the following is the resulting function?

$$y = \sqrt{6x-3} - 4$$

a) $y = \sqrt{3x+8} - 4$

b) $y = \sqrt{x-4} - 4$

c) $y = \sqrt{x+4} - 4$

d) $y = \sqrt{x+9} - 4$

e) *None of the above*

$y = \sqrt{6x-3} - 4$

① H.S. 1L $\quad x \to x+1 \quad y = \sqrt{6(x+1)-3} - 4 \Rightarrow y = \sqrt{6x+3} - 4$

② H.E. by 2 $\quad x \to \frac{1}{2}x \quad y = \sqrt{6(\frac{1}{2}x)+3} - 4 \Rightarrow y = \sqrt{3x+3} - 4$

③ H.S. 2L $\quad x \to x+2 \quad y = \sqrt{3(x+2)+3} - 4 \Rightarrow y = \sqrt{3x+9} - 4$

④ H.E. by 3 $\quad x \to \frac{1}{3}x \quad \boxed{y = \sqrt{x+9} - 4}$ ✓

The graph of $y = -2f(x+5)$ is the same as the graph of

H.S. 5L $x \to x+5$
V.R. and V.E. by 2 $y \to -\frac{1}{2}y$

**A.** The graph of $y = f(x)$ reflected about the $x$-axis, then shifted five units right, then stretched vertically by a factor of 2 about the $x$-axis.

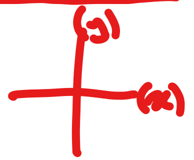
**B.** The graph of $y = f(x)$ reflected about the $y$-axis, then stretched vertically by a factor of $\frac{1}{2}$ about the $x$-axis, then shifted five units left.

**C.** The graph of $y = f(x)$ stretched by a factor of 2 about the $y$-axis, reflected about the $y$-axis, then shifted five units left.

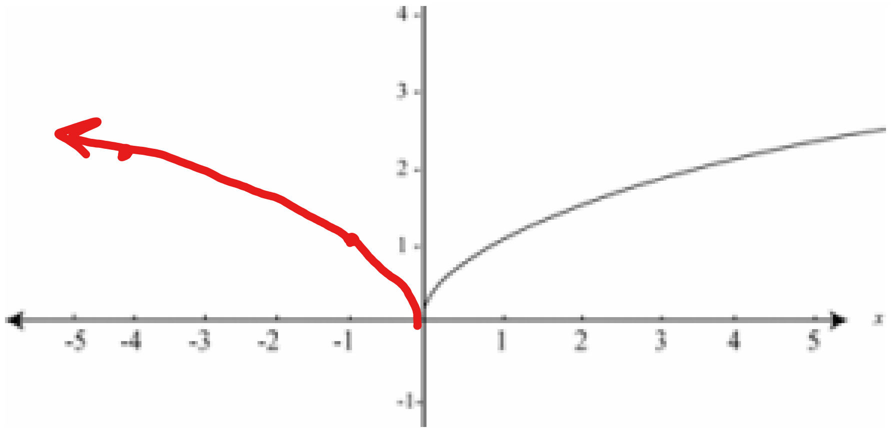
**D.** The graph of $y = f(x)$ stretched by a factor of 2 about the $x$-axis, reflected about the $x$-axis, then shifted five units left.

V.E.

V.R.



The graph of $f(x) = \sqrt{x}$ is shown below




The statement which best describes the graph of $g(x) = f(-x)$ is

H.R. over y-axis

D: $x \leq 0$

R: $y \geq 0$

~~A.~~ $g(x)$ is defined for all values of $x$

~~B.~~ $g(x)$ is defined for $x \geq 0$

C. $g(x)$ has a range of $y \geq 0$

~~D.~~ $g(x)$ is undefined for all values of $x$

The point (8, -5) is on the graph of $y = f(x)$. If the transformation $y = f(2x+4)$ is applied, then the new point is

A. (2, -5)
B. (20, -5)
C. (0, -5)
D. (4, -1)

Approach 1

H.S. 4L $x \to x+4$ $y=f(x+4)$

H.C. by $\frac{1}{2}$ $x \to 2x$ $y=f(2x+4)$

$(a,b) \to (a-4,b) \to \left(\frac{a-4}{2}, b\right)$

$(8,-5) \to (4,-5) \to \boxed{(2,-5)}$

Approach 2

H.C. by $\frac{1}{2}$ $x \to 2x$ $y=f(2x)$

H.S. 2L $x \to x+2$ $y=f(2(x+2))$

$\hookrightarrow y=f(2x+4)$

$(8,-5) \to (4,-5) \to \boxed{(2,-5)}$

The following function will all undergo a horizontal expansion by a factor of 2. Which of the following is the correct equation after the transformation?

$$y = 2^{x+1}$$

H.E. by 2 $\quad x \to \frac{1}{2}x \quad y = 2^{\frac{1}{2}x+1}$

$\Rightarrow y = 2^{0.5x+1}$

$\Rightarrow y = 2^{0.5x} \times 2^1 = 2 \cdot 2^{0.5x}$

$\boxed{(d)}$

$$a)\, y = 2^{2x+1}$$

$$b)\, y = 2^{2x+2}$$

$$c)\, y = 2^{0.5x+0.5}$$

$$d)\, y = 2 \times 2^{0.5x}$$

The following function will all undergo a horizontal expansion by a factor of 3. Which of the following is the correct equation after the transformation?

$$y = \sqrt{3x+2}$$

$a) y = \sqrt{x+2}$

H.E. by 3 $\quad x \to \frac{1}{3}x \quad \boxed{y = \sqrt{x+2}}$

$b) y = \sqrt{9x+2}$

$c) y = \sqrt{\frac{1}{3}(3x+2)}$

$d) y = \sqrt{3(3x+2)}$

The graph of $y = \sqrt{x}$ is vertically stretched by a factor of 2 about the $x$-axis, then reflected about the $y$-axis, and then horizontally translated left 3. What is the equation of the transformed function?

H.R. over y-axis

**A** $y = 2\sqrt{-x-3}$

**B** $y = 2\sqrt{-x+3}$

**C** $y = -2\sqrt{x+3}$

**D** $y = -2\sqrt{x-3}$

V.E. by 2 $\quad y \to \frac{1}{2}y \quad y = 2\sqrt{x}$

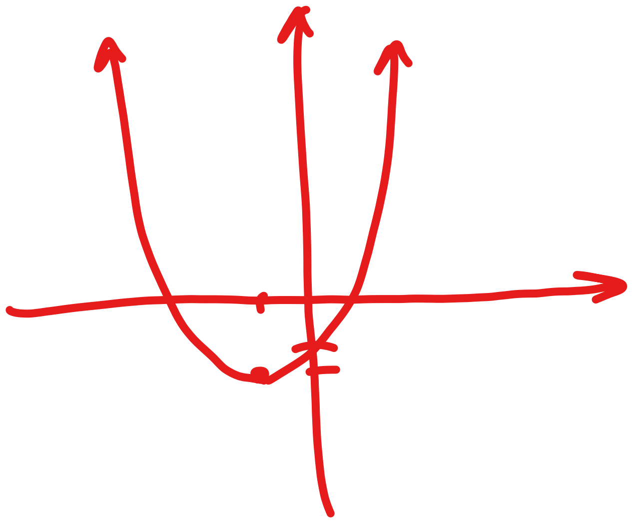
H.R. $\quad x \to -x \quad y = 2\sqrt{-x}$

H.S. 3L $\quad x \to x+3 \quad y = 2\sqrt{-(x+3)} = 2\sqrt{-x-3}$

(A)//

If the graph of $f(x) = x^2$ is transformed to the graph of $y + 2 = f(x+1)$, then a true statement regarding the two graphs is

A. The domain, but not the range, is the same.
B. The range, but not the domain, is the same.
C. Both the domain and range are the same
D. The domain and range are both different



$y = x^2 \longrightarrow y = (x+1)^2 - 2$

$D: x \in \mathbb{R} \longrightarrow D: x \in \mathbb{R}$

$R: y \geq 0 \longrightarrow R: y \geq -2$

Which of the following transformations would produce a graph with the same $x$-intercepts as $y = f(x)$?

**A** $y = -f(x)$

**B** $y = f(-x)$

**C** $y = f(x + 1)$

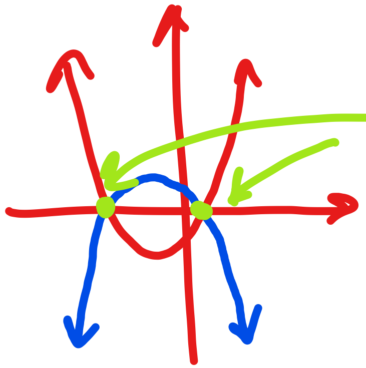
**D** $y = f(x) + 1$

When performing a V.R. over $x$-axis, our $x$-intercepts stay the same.

$$y = f(x) \xrightarrow[\text{V.R.}]{} y = -f(x)$$

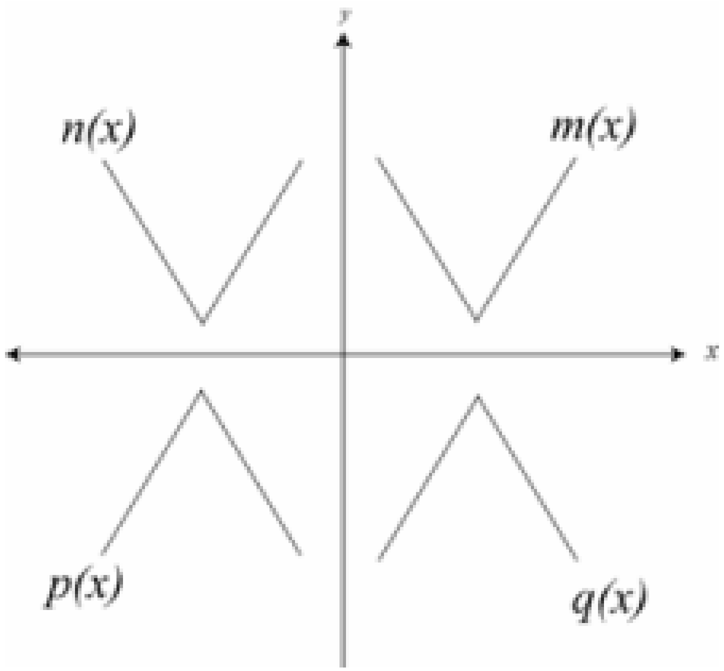
$0 \quad x - 1 = 0$

$(r, 0) \rightarrow (r, 0)$



Stays the same!

The graph of $m(x)$ is shown, along with three possible reflections.




A student knows the following reflections were used:

1. $y = -f(x)$
   V.R. over x-axis
2. $y = f(-x)$
   H.R. over y-axis
3. $y = -f(-x)$
   V.R & H.R

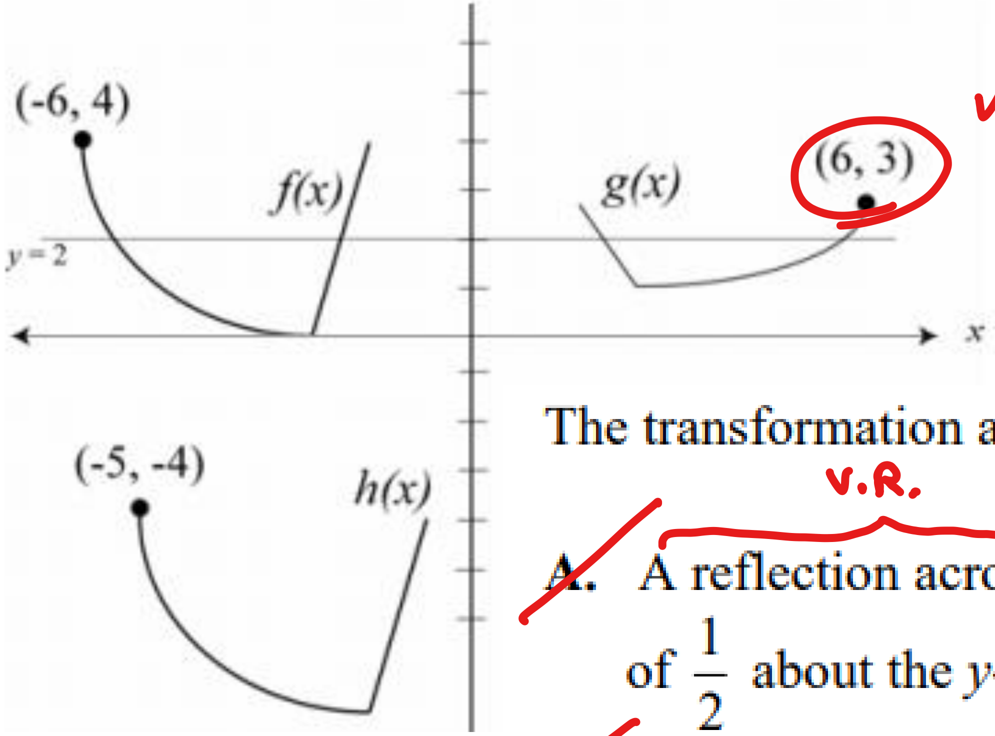
Which equation is n(x), p(x) , and q(x)??

1: $q(x)$

2: $n(x)$

3: $p(x)$

The graphs of $f(x)$, $g(x)$, and $h(x)$ are shown below




The transformation applied to $f(x)$ in order to obtain $g(x)$ is

V.R.

~~A.~~ A reflection across the $x$-axis, then a vertical stretch by a factor of $\frac{1}{2}$ about the $y$-axis.

~~B.~~ A reflection across the $y$-axis, then a vertical stretch by a factor of $\frac{1}{2}$ about the $x$-axis.

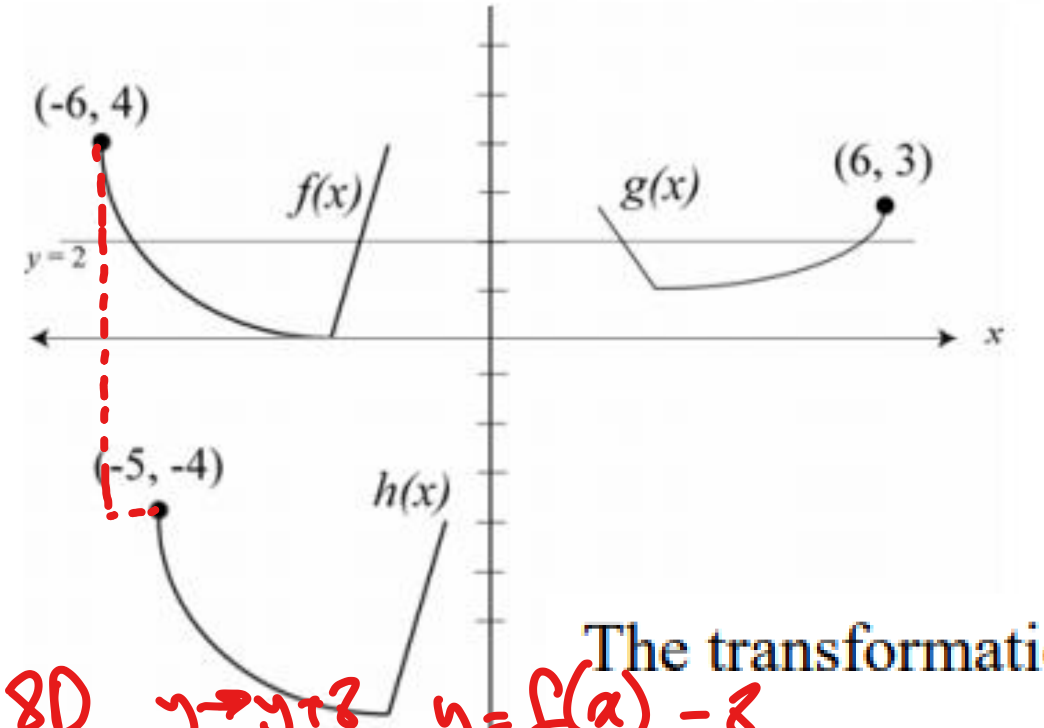
C. A vertical stretch by a factor of 2 about the line $y = 2$, then a reflection across the $y$-axis.

**D.** A vertical stretch by a factor of $\frac{1}{2}$ about the line $y = 2$, then a reflection across the $y$-axis.

everythg gets compressed towards $y=2$ by factor of $\frac{1}{2}$.

$(-6,4) \rightarrow (-6,3) \rightarrow (6,3)$ //

The graphs of $f(x)$, $g(x)$, and $h(x)$ are shown below




The transformation applied to $f(x)$ in order to obtain $h(x)$ is

V.S. 8D $y \to y+8$ $y = f(x) - 8$

H.S. 1R $x \to x-1$ $y = f(x-1) - 8$

**A.** $h(x) = -f(x-1) - 8$

**B.** $h(x) = f(x-1) - 8$

**C.** $h(x) = f(x+1) + 8$

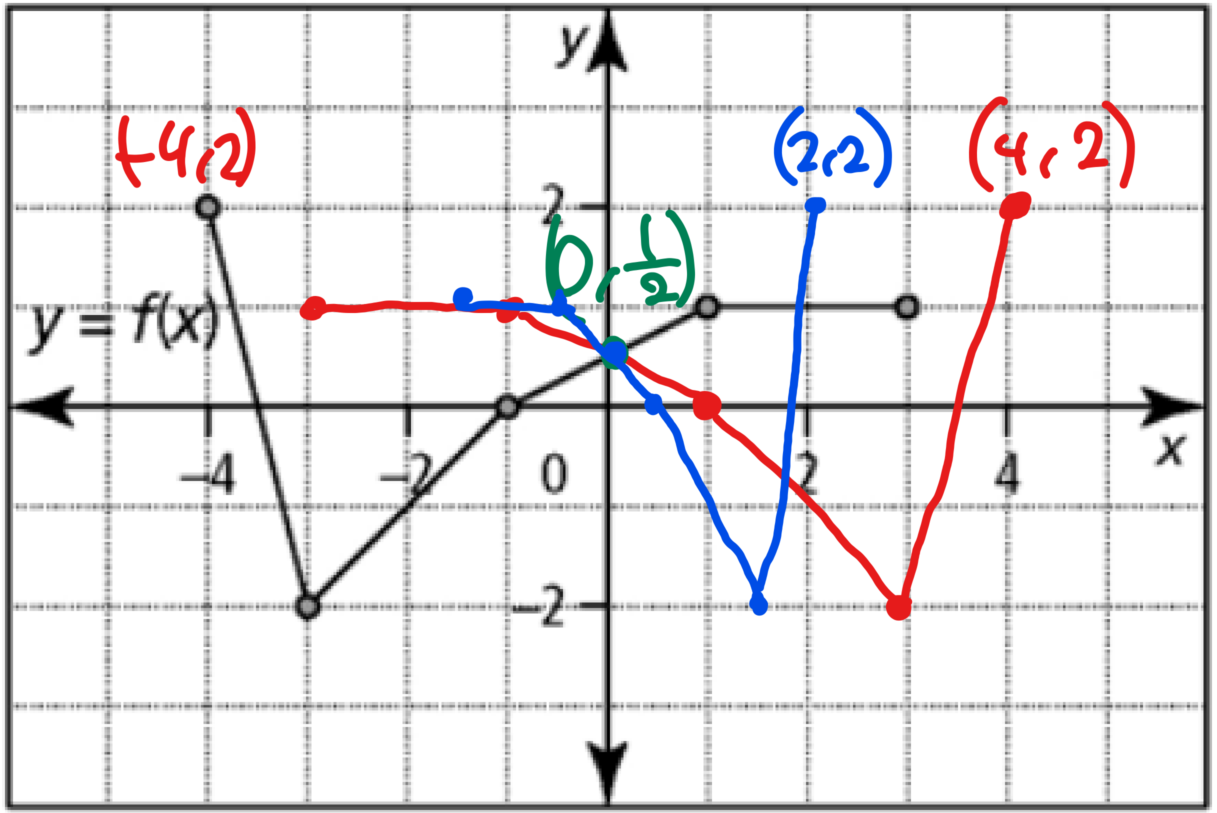
**D.** $h(x) = f(x+1) - 8$

. Given the graph of $y = f(x)$, what is the invariant point

under the transformation $y = f(-2x)$?

H.R. over y-axis

H.R. and H.C. by $\frac{1}{2}$




y-intercepts don't change

$(-4,2) \to (2,2)$

$(0, \frac{1}{2}) \to (-\frac{1}{2} \times 0, \frac{1}{2}) = (0, \frac{1}{2})$

invariant!

**A** $(-1, 0)$ **B** $(0, \frac{1}{2})$ **C** $(1, 1)$ **D** $(3, 1)$

The graph of $y = f(x)$ is horizontally stretched by a factor of 3 about the *y*-axis, reflected in the *x*-axis, then translated four units right and two units up.
The transformed graph is represented by

A. $y = -f\left(\frac{1}{3}(x-4)\right)+2$

B. $y = -f(3(x-4))+2$

C. $y = f(-3(x-4))+2$

D. $y = f\left(\frac{1}{3}(-x-4)\right)+2$

① H.E. by 3 $\quad x \to \frac{1}{3}x \quad y = f\left(\frac{1}{3}x\right)$

② H.S. 4R $\quad x \to x-4 \quad y = f\left(\frac{1}{3}(x-4)\right)$

③ V.S. 2U $\quad y \to y-2 \quad \boxed{y = f\left(\frac{1}{3}(x-4)\right)+2}$

A function $f(x)$ is transformed to produce the graph of $g(x) = f(x-7)+8$. If the graph is further transformed by moving it two units left and one unit down, then the new graph can be written as $h(x) = f(x-a)+b$. The numerical values of $a$ and $b$ are, respectively, __5__, and __7__.

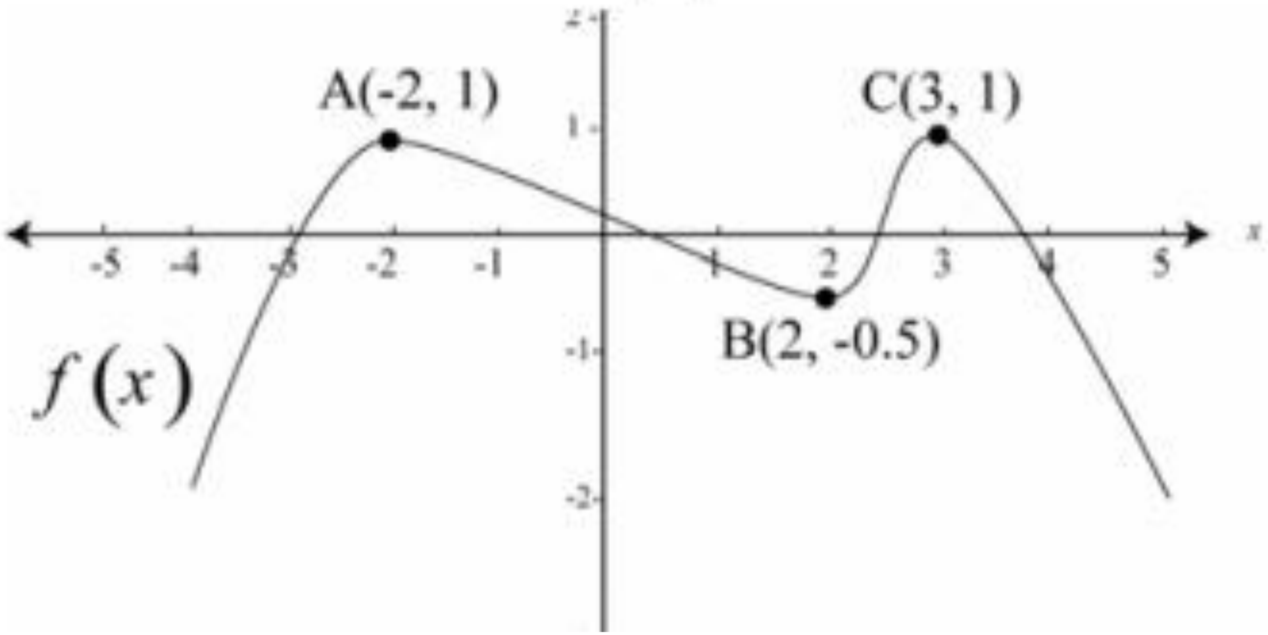
$$g(x) = f(x-7)+8$$

H.S. 2L $\quad x \to x+2$ $\qquad g(x) = f(x-5)+8$

V.S. 1D $\quad y \to y+1$ $\qquad g(x) = f(x-5)+7$

$$h(x) = f(x-5)+7 \; ; \; a=5, \; b=7$$

Points on the graph of $y = f(x)$ are shown below




If the graph is transformed to $g(x) = f(2x-4)$, then point **A** becomes $(m, 1)$. The value of $m$ is

H.S. 4R $x \to x-4$ A(2,1)

H.C. by $\frac{1}{2}$ $x \to 2x$ A(1,1)

$\boxed{m=1}$

**A.** 0

**B.** 1

**C.** 3

**D.** 4

The domain of $f(x)$ is $x \leq 3$. If the transformation $g(x) = f(x+10) - 2$ is applied, then the new domain of the function is

H.S. 10L

V.S. doesn't change domain

A. $x \leq -10$
B. $x \leq -7$
C. $x \geq -10$
D. $x \geq -7$

$x \leq 3 \rightarrow x \leq -7$

A point on the graph of $f(x)$ is (-3, 4). If the transformation $y = f(3x-6)-1$ is applied, then the new coordinates of the point are

① H.S. 6R $x \to x-6$

② H.C. by $\frac{1}{3}$ $x \to 3x$

③ V.S. 1D $y \to y+1$

$(-3,4) \xrightarrow{①} (3,4) \xrightarrow{②} (1,4) \xrightarrow{③} (1,3)$

**A.** (1, 3)
**B.** (-1, 4)
**C.** (-15, 3)
**D.** (5, 3)

The function $f(x) = x^2 - 5x + 6$ is multiplied by a constant $b$ to apply a vertical stretch to the graph. If the transformed graph passes through the point (8, 15), then the value of $b$ is _________.

**A.** 4

**B.** $\frac{1}{4}$

**C.** 2

**D.** $\frac{1}{2}$

$f(x) = bx^2 - 5bx + 6b$

$b(8)^2 - 5b(8) + 6b = 15$

$b(64 - 40 + 6) = 15$

$\boxed{b = \frac{15}{30} = \frac{1}{2}}$

A function $f(x) = x^2 - x - 2$ is multiplied by a constant value $a$ to create a new function $g(x) = af(x)$. If the graph of $y = g(x)$ passes through the point (3, 14), state the value of $a$.

$g(x) = a(x^2 - x - 2)$

$g(3) = 14$

$\therefore \quad 14 = a(3^2 - 3 - 2) \Longrightarrow \boxed{a = \frac{14}{4} = \frac{7}{2}}$

If the graph of f(x) undergoes the transformation $y = f(\frac{1}{5}x)$, a point that exists on the graph of the image is:

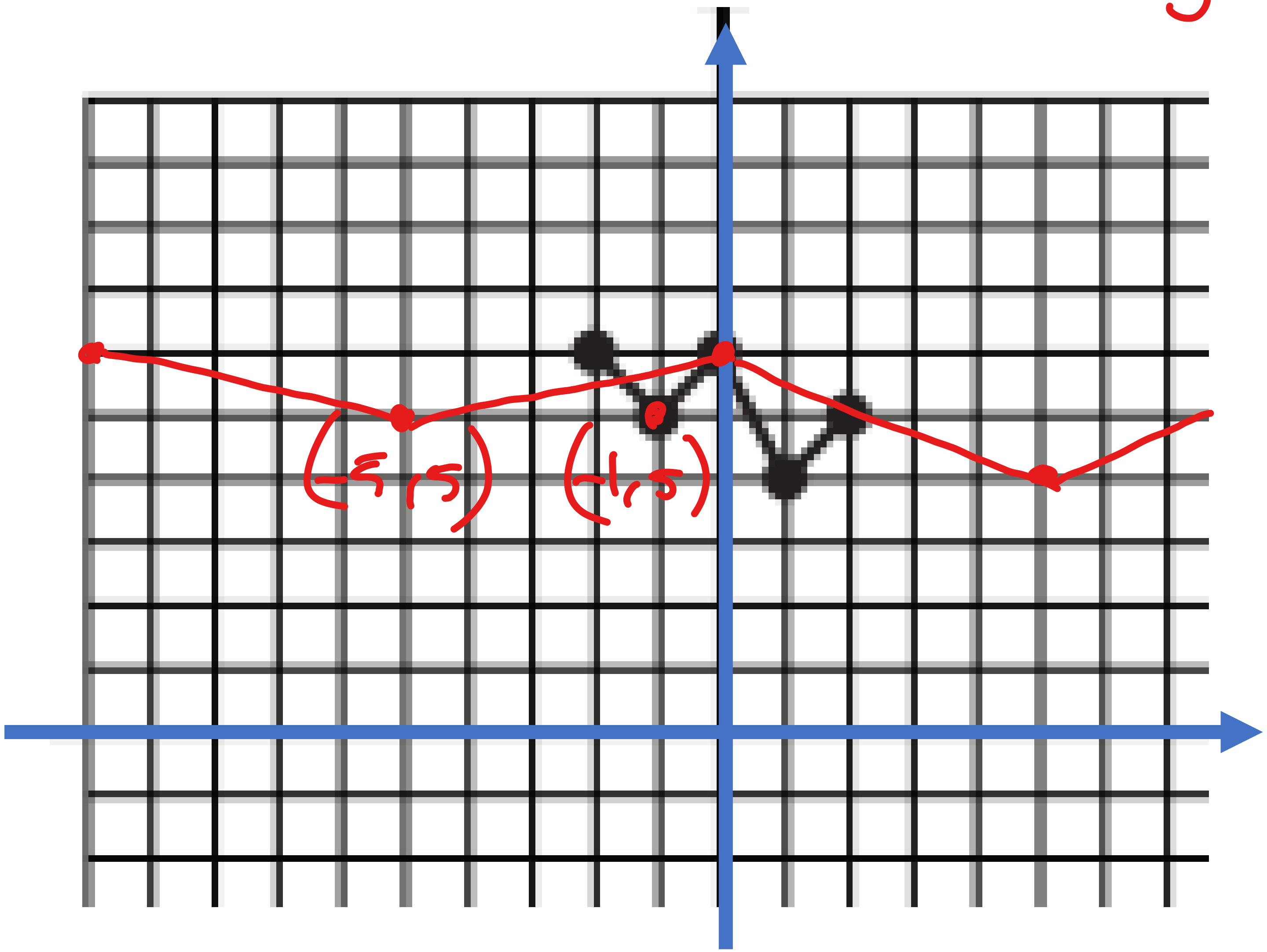
H.E. by 5

A. $\left(\frac{1}{5}, 4\right)$

B. (2, 1)

C. (-5, 5)

D. (6, 0)



If the graph of f(x) undergoes the transformation y - 4 = f(x), then the range of the image is:

V. S. 4U

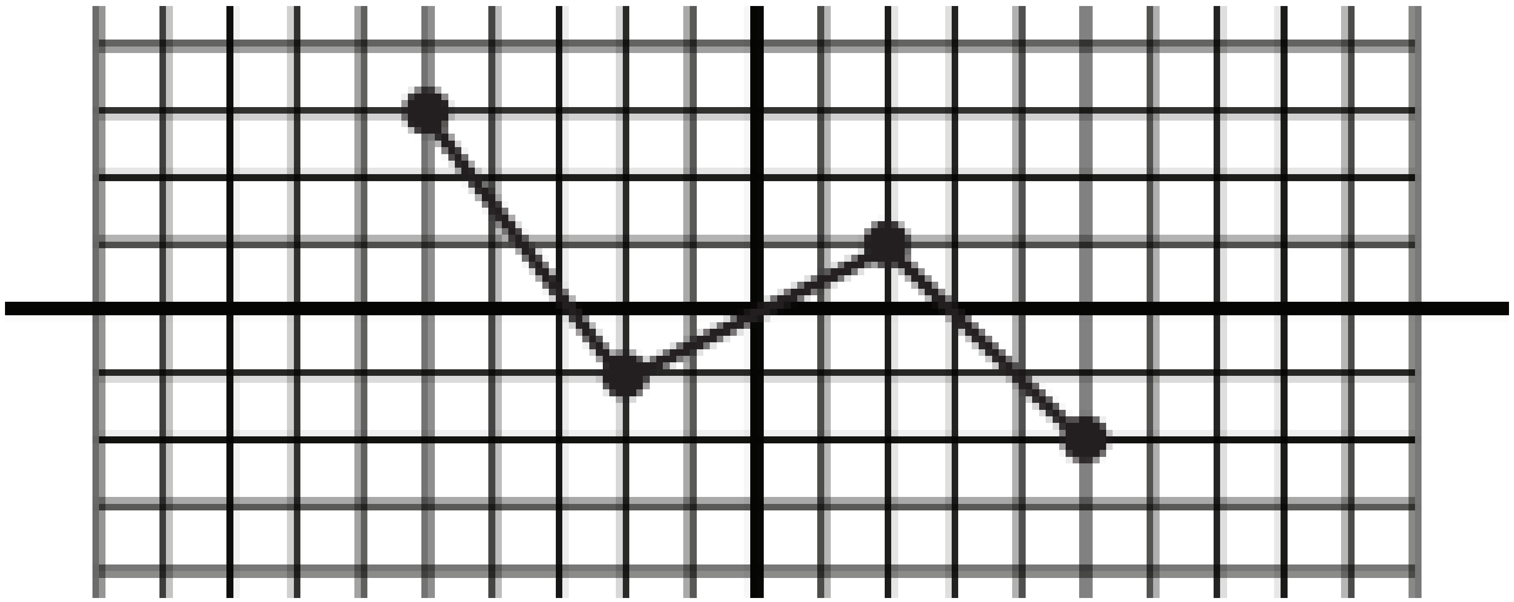
A. {y | -6 ≤ y ≤ -1, y ε R}

B. {y | 2 ≤ y ≤ 7, y ε R}

not true

C. [-6, -1]

D. ~~(2, 7)~~



R: $-2 \leq y \leq 3$

V.S. 4U

new R: $\boxed{2 \leq y \leq 7}$ ✓

As a result of the transformation of the graph of $y = f(x)$ into the graph of $y = -3f(x + 2) - 5$, the point (2, 5) becomes point $(x, y)$. Determine the value of $(x, y)$.

$y = f(x)$

① H.S. 2L $\quad x \to x+2 \quad y = f(x+2)$

② V.R. and V.E. by 3 $\quad y \to -\frac{1}{3}y \quad y = -3f(x+2)$

③ V.S. 5D $\quad y \to y+5 \quad y = -3f(x+2) - 5$

$(2, 5)$

$(0, 5)$

$(0, -15)$

$(0, -20)$

$\boxed{(x, y) = (0, -20)}$ ✓

The general transformation equation y = af[b(x - h)] + k can be expressed as the mapping:

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

**Legend**

VS - Vertical Stretch
VR - Reflection About the x-axis
VT - Vertical Translation

HS - Horizontal Stretch
HR - Reflection About the y-axis
HT - Horizontal Translation

Based on the mapping, one can conclude that:

✓ **A.** Transformations are axis-independent.
The transformation sequence [VS - VR - VT - HS - HR - HT] is correct because all vertical transformations are grouped together and all horizontal transformations are grouped together.

✗ **B.** Stretches and reflections must universally be applied before translations.
The transformation sequence [VS - VR - VT - HS - HR - HT] is incorrect because a vertical translation is applied before a horizontal stretch.

✓ **C.** Stretches and reflections can be applied in either order since the negative sign is included in the *a* and *b* parameters. The transformation sequence [VR - VS - VT - HR - HS - HT] is correct.

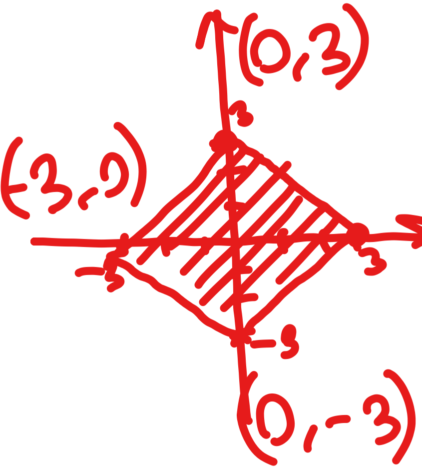
**D.** Both A and C are correct.

My response may be incorrect for this one.

What is the area of the region defined by the inequality $|3x - 18| + |2y + 7| \leq 3$?

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 5

$|x| + |y| \leq 3$




$x + y \leq 3 \Rightarrow y = 3 - x$

Abs. value type IV

$\Rightarrow |y| = 3 - |x|$

① H.S. 18R $\quad x \to x - 18 \quad |x-18| + |y| \leq 3$

② H.C. by $\frac{1}{3}$ $\quad x \to 3x \quad |3x-18| + |y| \leq 3$

③ V.S. 7D $\quad y \to y+7 \quad |3x-18| + |y+7| \leq 3$

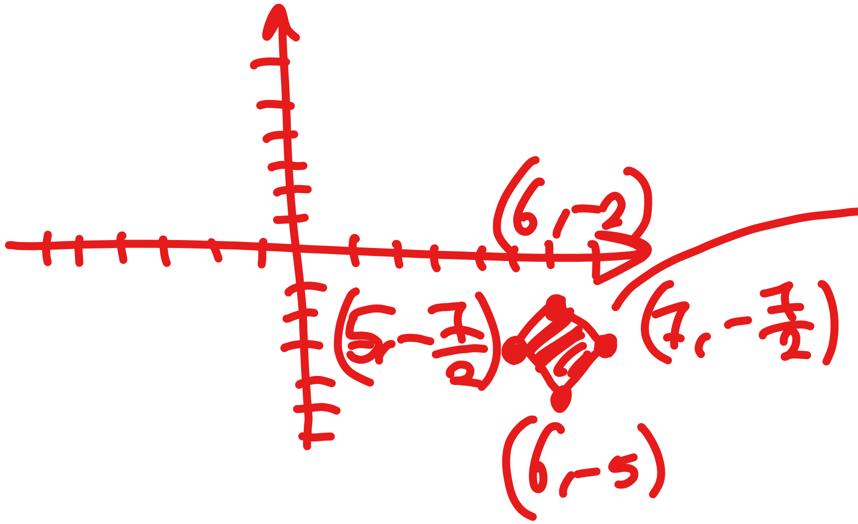
④ V.C. by $\frac{1}{2}$ $\quad y \to 2y \quad |3x-18| + |2y+7| \leq 3$

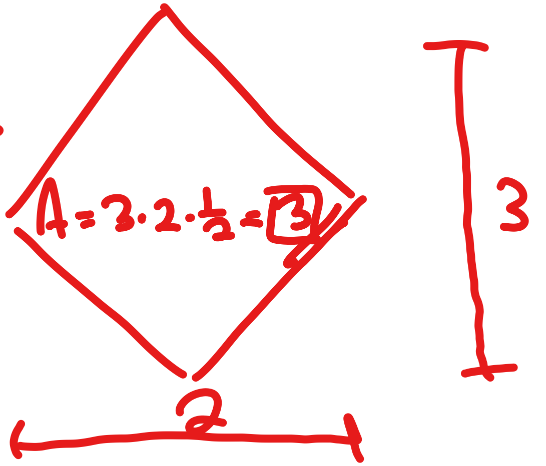
$(0,3) \to (6,3) \to (6,-2)$

$(0,-3) \to (6,-3) \to (6,-5)$

$(3,0) \to (7,0) \to (7,-\frac{7}{2})$

$(-3,0) \to (5,0) \to (5,-\frac{7}{2})$



Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2,1)$ is $A'(2,-1)$ and the image of $B(-1,4)$ is $B'(1,-4)$?

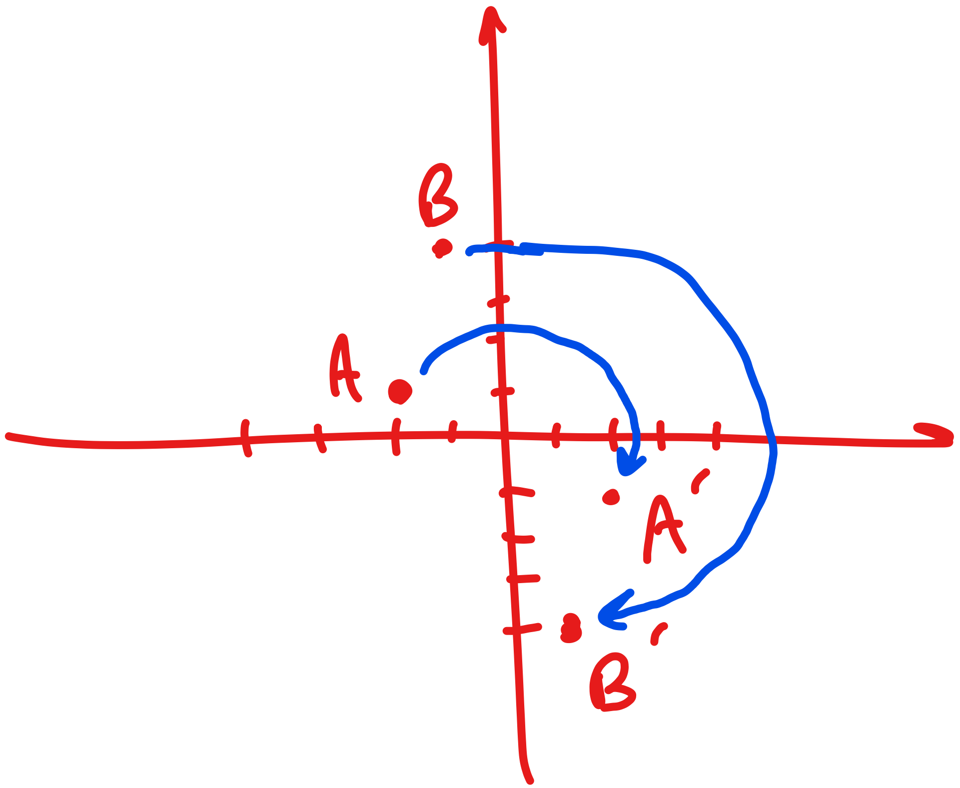
**(A)** reflection in the $y$-axis

**(B)** counterclockwise rotation around the origin by $90°$

(C) translation by $3$ units to the right and $5$ units down

**(D)** reflection in the $x$-axis

**(E)** clockwise rotation about the origin by $180°$

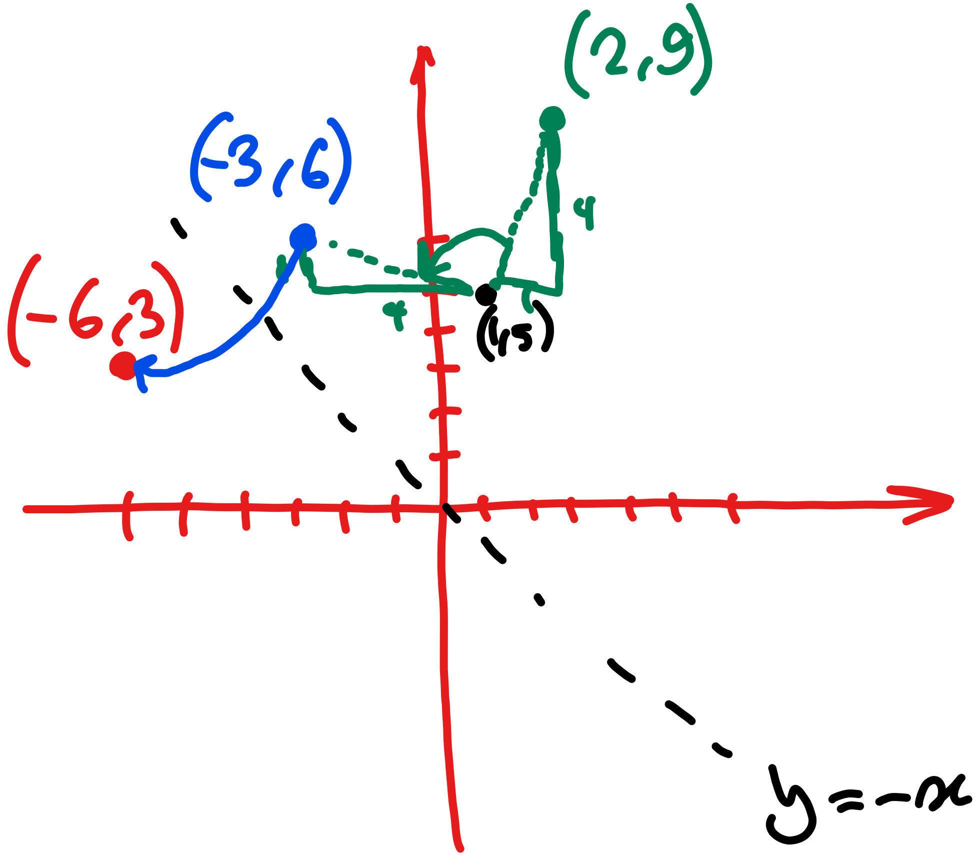


The point $P(a, b)$ in the $xy$-plane is first rotated counterclockwise by $90°$ around the point $(1, 5)$ and then reflected about the line $y = -x$. The image of $P$ after these two transformations is at $(-6, 3)$. What is $b - a$?

**(A)** 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

$(a, b) = (2, 9)$

$b - a = 9 - 2 = \boxed{7}$



## Problem 12

Line $\ell$ in the coordinate plane has the equation $3x - 5y + 40 = 0$. This line is rotated $45^\circ$ counterclockwise about the point $(20, 20)$ to obtain line $k$. What is the $x$-coordinate of the $x$-intercept of line $k$?

$\textbf{(A)}\ 10 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 20 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 30$

Let $T$ be the triangle in the coordinate plane with vertices $(0,0)$, $(4,0)$, and $(0,3)$. Consider the following five isometries (rigid transformations) of the plane: rotations of $90^\circ$, $180^\circ$, and $270^\circ$ counterclockwise around the origin, reflection across the $x$-axis, and reflection across the $y$-axis. How many of the $125$ sequences of three of these transformations (not necessarily distinct) will return $T$ to its original position? (For example, a $180^\circ$ rotation, followed by a reflection across the $x$-axis, followed by a reflection across the $y$-axis will return $T$ to its original position, but a $90^\circ$ rotation, followed by a reflection across the $x$-axis, followed by another reflection across the $x$-axis will not return $T$ to its original position.)

**(A)** 12 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 25

## Problem 19

Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1), B(-1,1), C(-1,-1),$ and $D(1,-1)$. Consider the following four transformations:

- $L$, a rotation of $90^\circ$ counterclockwise around the origin;
- $R$, a rotation of $90^\circ$ clockwise around the origin;
- $H$, a reflection across the $x$-axis; and
- $V$, a reflection across the $y$-axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying $R$ and then $V$ would send the vertex $A$ at $(1,1)$ to $(-1,-1)$ and would send the vertex $B$ at $(-1,1)$ to itself. How many sequences of $20$ transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, $R, R, V, H$ is one sequence of $4$ transformations that will send the vertices back to their original positions.)

$\textbf{(A)}\ 2^{37} \qquad \textbf{(B)}\ 3 \cdot 2^{36} \qquad \textbf{(C)}\ 2^{38} \qquad \textbf{(D)}\ 3 \cdot 2^{37} \qquad \textbf{(E)}\ 2^{39}$